# PHYSICS Unit 4 Reading: Forces and Equilibrium

In this chapter we begin our study of forces. **Forces** are interactions between objects that result in a push or a pull. Before we study forces, we will first learn how to determine the resultant of vectors that are not perpendicular. The resultant of force vectors is the value of a single force that could replace all of the forces acting on the object. We then study forces on an object in a condition called equilibrium. **Equilibrium** is a state where an object is not accelerating. A special type of equilibrium is **static equilibrium** where an object is not moving. The unit of force we will use is the newton (N). The definition of the newton will wait until unit 5.

#### **Part 1: Newton’s First Law**

An object at rest, or in motion at a constant speed in a straight line (in other words at a constant velocity), will remain in that state unless acted on by a net unbalanced force. In other words, if the forces on an object add up to zero, the object will not accelerate. It may not move at all, or it may move at a constant velocity. Both of these are states of equilibrium. This means that if an object is moving at a constant velocity, there are either no forces acting on the object, or the forces that are acting on it must add up to zero.

Matter has a property called **inertia**, which is defined as the resistance of an object to a change in motion. It is a property that depends only on how much mass the object has. If a puck is given a push on an air table, it will keep moving after it leaves contact with the hand. When asked what force causes the puck keeps going, never say the force of inertia. Inertia is not a force; it is just a resistance to a change in motion. The puck keeps going because there is no force causing it to slow down.

## Part 2: Addition of vectors

**A: Graphical addition**

In chapter 9 you learned the basics of graphical vector addition. We will now use the method on more difficult problems. To review, the steps for graphically adding vectors are:

1. Draw one of the vectors in the problem to scale and point it in the correct direction.

2. Draw the next vector, starting with it’s tail at the tip of the first vector.

3. If there are more than two vectors, draw each with the tail starting at the tip of the previous. Continue the process until all of the vectors in the problem have been drawn.

4. Draw in the resultant, starting at the tail of the first vector and ending at the tip of the last.

5. Measure the length and angle of the resultant to get the value. You are always to measure the angle from an axis placed at the tail of the first vector.

**Sample Problems**:

1) What is the resultant of the following two forces, 15 N east and 25 N at 30.° E of N?



R = 7.0 cm x 5 N/cm = 35 N

θ = 38° N of E

R

1 cm = 5 N

2) Three boys are pulling on Jane; Jim with a force of 50.N to the west, John with a force of 80.N at 45° S of W, and Jack with a force of 120 N at 60.° N of E. What is the resultant force acting on Jane?



R = 3.30 cm x 20 N/cm = 66 N

θ = 43o W of N

θ

R

1 cm = 20 N

**B: The component method of vector addition**

In chapter 9 you learned how to find the components of a vector. Now this skill will be used to add vectors together that are not at right angles. First, we need to take a look at our coordinate system. When vectors are added in the component method, a positive or negative value is given to each component based on the direction the component is pointing. As shown in the diagram, north (or up) and east (or right) are positive, and south (or down) and west (or left) are negative.

E = +

N =+

S = -

W = -

**Steps for addition of vectors using the component method.**

1. Draw a sketch with the tail of each vector starting at the origin on the coordinate system. Identify the angles of each vector as measured from horizontal.

2. Using the angles from horizontal, calculate the horizontal components of each vector using the cosine function, and the vertical components using the sine function. Make sure the signs (+ or -) are correct.

3. Add all of the horizontal components together, and then add all of the vertical components together. Those two numbers are the components of the resultant vector.

4. Take your horizontal and vertical values, and use them to make a right triangle.

5. Use the Pythagorean theorem to obtain the magnitude and the tangent function to obtain the angle of the resultant.

**Sample Problems**:

3) What is the resultant of the forces 15 N east and then 25 N at 30o E of N?

x components

(horizontal)

cos0o(15 N) = 15 N

cos60o(25 N) = 12.5 N

Total 27.5 N

y components

(vertical)

sin0o(15 N) = 0 N

sin60o(25 N) = 21.65 N

Total 21.65 N

60o

15 N

30o

25 N

27.5 N

21.65 N

R

Remember to measure the angles from horizontal!!



## Part 3: Forces

As stated earlier a force is an interaction between two objects that results in a push or a pull. Forces can be categorized as contact forces or forces that act at a distance. In contact forces the two objects must be touching each other. Often one can see evidence of a contact force by objects bending, stretching or compressing. Common examples of contact forces are friction, tension and normal forces. Examples of forces at a distance are the force of gravity, magnetic forces and electric forces. When objects are able to exert a force at a distance, we say the object has a field (gravitational field, magnetic field, and electric field) around it. The forces on the block in the diagram below will be used to introduce some of the more common forces.

**Force of gravity (Fg or FW)**: the attractive force between all objects. In this case the earth is pulling the block toward it. This force is also called **weight**. The direction of the weight force is **always** down.

**Normal (FN)**: the force exerted by a surface of one object on another object. The direction of a normal force is perpendicular to the surface. The block is being pushed upward by the counter top.

**Tension (FT)**: the forces exerted by ropes, strings, cables, etc., on an object they are connected to. The direction of the tension is in the direction the string is pulling. The block is being pulled to the right by the string. For our work, the amount of tension is the same everywhere in the string from one end to the other.

**Friction (Ff)**: when one object is sliding relative to another object while their surfaces are touching, we call their scraping, the force of kinetic friction. Kinetic friction () is present if the block is sliding across the counter. It is the counter is pushing to the left on the block. When a force is exerted on an object and it is not able to move due to the engagement of the surfaces, that force is called static friction (). If the string is pulling to the right but the block doesn’t move, there is static friction to the left.

##### Equilibrium

We will get into Newton’s laws in great detail in Unit 5, but will apply his first law at this time. To paraphrase the first law: **If an object is not in accelerating, the vector sum of the forces acting on the object is zero.** Another phrase used instead of vector sum is to call it the **net force**. If the net force is zero, the forces on the object are said to be **balanced**, and if they do not add up to zero, the forces are said to be **unbalanced**. So, if the object is not accelerating the net force is zero, and the forces are balanced.

### Force diagrams

A force diagram (sometimes called a free body diagram) is a device we use to show the forces acting on an object, the direction they are acting and in some cases the relative size of the force. The steps we will use are shown below for the block in the diagram above as it slides across the counter at a constant velocity:

Force diagram for the block above as is moves across the counter at a constant velocity

1. Draw a rough outline of the object in question.
2. Place an arrow on your outline for each force acting on the object. Point the arrow in the direction of the force. Label the arrow with the type of force, the object causing the force and the object the force is acting on. On the block, the label on the upward arrow represents the normal force of the counter on the block. The arrow to the right represents the tension force of the string on the block. The arrow to the left represents the kinetic friction force of the counter on the block. The arrow down represents the force of gravity of the earth on the block. Every label should end with the same letter.



FNcb

Fgeb

FTsb

1. If the object is in equilibrium, the arrows should be sized so they add up to zero. If the object is accelerating, the object will have a net force in the direction of the acceleration. We were told the block was sliding at a constant velocity, so the left and right arrows are of the same size. The block is not moving up or down so those two arrows are also of the same size.

### Objects on an incline

Objects on an incline present a more difficult situation because of the orientation of the forces. Diagram *a* shows a force diagram for a block at rest on an incline. There are three forces exerted on the block, the earth is pulling it down (weight), the incline is pushing it perpendicular to the surface (normal) and pushing it parallel to the surface (friction). We know the all of the forces are balanced because the block is not accelerating. Where are the forces balancing the friction and the normal force? The weight force can be split into two components. One component is the part of the weight that is pulling the block down the plane. This component will be called F⏐⏐, for force parallel to the surface of the plane. This component balances the friction force. The other component is the part of the weight pulling the block into the plane. This component will be called F⊥, for force perpendicular to the plane. This perpendicular component balances the normal force. The force diagram reflects this in that if the three vectors are added together, they will produce a resultant of zero.

diagram *a*

FNib



Fgeb

On diagram *b* the force diagram is redrawn with the weight vector broken into the parallel and perpendicular components. Note that our coordinate system has been rotated so the x-axis is parallel to the plane and the y-axis is perpendicular. The angle of the plane to horizontal is the same as the angle between the weight vector and the y-axis. To prove this, look at diagram *c*.



y

Fgy

θ

x

Fgx

diagram *b*

Fgeb

FNib

To solve for numerical values we use trigonometry as shown on the triangle below.

1. The parallel component of the weight is the opposite side of the weight triangle, therefore **Fgx = sinθFg.**
2. The perpendicular component of the weight is the adjacent side of the weight triangle therefore, **F**gy **= cosθFg.**

Fgy **= cosθFg**

θ

Fgx = **sinθFg**

Fg





Fgx

W

θ

90-θ

θ

diagram *c*

###### Problem solving for equilibrium force problems

Now that you are able to identify forces on an object, we start solving for their numerical values. In the mathematical problems you are allowed to abbreviate the forces acting on an object. Where tension is involved, instead of something like **FTsb**, you are allowed to put a **T**. For the force of gravity, **Fgeb**, you are allowed to use a **W** or **Fg**. For the force of friction, , use an **f** or **fk** and for the normal force, **FNcb**, use an **N**. Before we start problem solving, we need to review the weight force as in some problems you will be given the mass of an object and need to be able to obtain its weight.

### Weight Force

Early this year we did a lab to determine the relationship mass and weight. We found the weight was equal to the mass in kilograms times the gravitational field strength in newtons/kilogram, which we call “g”. We determined the field strength to be  on the earth’s surface. To most people the terms mass and weight are interchangeable. They are not the same. **Mass** is the amount of matter an object contains. The standard unit of mass is the kilogram (kg). The weight force is caused by one mass pulling on another mass. This pull is caused by something we call gravity. We call the pull or force of gravity **weight**. Since the amount of material in most objects does not change, when the object is taken to places with a different pull of gravity, the mass does not change. However the weight does.

**Tip:** Read problems carefully. If the unit is N (newton) for the object, you are being given the weight or the force. If the unit is kg, you are being told the mass. If the unit is grams, it must be changed to kilograms to get the weight.

#### W = mg weight = mass times gravitational field strength

**Sample Problem**:

5) What is the weight of Grace Full, the 60.0 kg gymnast?

**Solution:**

W = mg = 60.0 kg

**Knowns:**

mass = 60.0 kg

g = 

**Equilibrium problem solving strategy:**

1. If a picture of the problem is not provided, sketch one.

2. Locate the object specified or an object in the diagram that has more than one force acting on it.

3. Draw a force diagram for that object. Arrows are used to represent forces. Place a label at the end of the arrow, such as a **W** for weight, a **T** for tension, or an **N** for normal force at the end of the arrow. In simple force diagrams for solving problems you are not required to pay attention to the sizes of the arrows.

4. Place a coordinate system on the object. We will use positive for up or to the right, and negative for down or to the left.

1. Write the equation for the sum of the horizontal forces and set the sum equal to zero. Write the equation for the sum of the vertical forces and set the sum equal to zero.
2. Solve for your unknowns.

**Sample Problems**:

6) Write the equations and solve for the values of the normal force and the friction force for the 220 kg log being pulled at a constant speed by a rope with a tension of 440 N?

+

+

**Coordinate system**

In the equations the weight and friction forces were made negative because they are in the negative directions.

N



N - W = 0 or W = N T – fk = 0 or T = fk

N = W = mg = 220 kg T = 440 N

T

fk

W

7) What is the normal force on the bottom of each block?

W2

N12

N

N-W2-N12 = 0

or N= W2 +N12

N = 12 N + 2 N = 14 N

N21

W1

2.0N

N21 stands for normal of block 2 on block 1.

Equation:

12 N

N21 - W1 = 0

or N21 = W1 = 2.0 N

8) Kara Vann is standing on a hill inclined at 20.° above horizontal. She is keeping her brother from rolling down the hill. If her brother and tricycle weigh 200.N, how hard does she have to push? With how much force is the earth pushing on the tricycle?

+

+

**Force diagram**

**Coordinate system**

N = force earth

W

F⊥

F⏐⏐



Force Kara

**Solution:**

parallel equation: FKara - F⏐⏐ = 0

perpendicular equation: N - F⊥ = 0

FKara = F⏐⏐ = sinθW = sinθW = sin20°(200 N) = 68 N

N = F⊥ = cosθW = cosθW = cos20°(200N)=190 N

9) What is the tension in the top and bottom rope and the mass of W1?

W = 4N

T2

T2 – 4N = 0

T2 = 4.0 N

bottom

6.0 N

W1

T2

middle

6N – W1` – T2 = 0

two unknowns so we move on

T1

W=14N

T2 = 6.0N

T1 – 14N –6N = 0

T1 = 20.0 N

top

W1

14.0N

4.0N

T = 6.0N

Now that we know T2 go back to middle sphere

6N – W1` – 4N = 0

W1= 2.0 N

M1 = 

10) What is the tension in each rope if the pulley is “weightless” and the 15 N weight is being raised at a constant speed?

It is the same rope on both sides of the pulley, so the tension must be the same on both sides.

W1

T2

W1

T2

T1

T1

2T1 = T2 and since

T2 = W1 , 2T1 = W1

T1 = 15N/2 = 7.5 N

T2 – W1 = 0

T2 = 15N

11) What forces are needed to keep the objects below in equilibrium?

25 N

15 N

Fy

18 N

Fx

a)

Fy

25 N

15 N

Fx

18 N

horizontal: 15 N - Fx = 0 → Fx = 15 N

vertical: Fy + 18 N - 25 N = 0

so Fy = 7 N

(Don’t forget the weight of the object!)

12) Solve for the tension in each of the three ropes below.

60 N

50o

From now on instead of setting the forces equal to zero, it is faster to say left = right and up = down. From looking at the diagram it is easy to deduce that T3 has to equal 60. N.

Hor: cos50o(T2) = T1

Ver: sin50o(T1) = 60 N, so T1 = 78 N

and T1 = cos50o(78 N) = 50. N

T1

T2

T3

50o

13) What is the tension in the ropes holding up the 165 N sign below?

Eric’s

Pizza

38o

38o

Horizontal: cos38o(T2)-cos38o(T1) = 0

therefore T1 = T2

Vertical: sin38o(T1) + sin38o(T2) - 165 N = 0

which if T1 = T2 becomes

2sin38o(T1) = 165 N

T1 = T2 =134 N

T1

T2

165 N

38o

38o

14) Solve for T1 and T2.

50o

70o

T1

T2

125 N

Hor: cos50o(T2) = cos70o(T1) so T1 = 1.88 T2

Ver: sin50o(T2) + sin70o(T1) = 125 N

substitute in: sin50o(T2) + sin70o(1.88T2) = 125 N

2.53T2 = 125 N so T2 = 49 N

T1 = 1.88(49 N) = 93 N

#### Part 4: Newton’s Third Law

Whenever object A exerts a force on object B, object B exerts an equal and opposite force back on A. Forces always occur in pairs, and the two forces always have the same magnitudes. The force of A on B is called the **action** force, and the force of B back on A is called the **reaction** force.

I think the best way to begin to develop an understanding of the laws is to pose some questions and use Newton’s Laws to answer the questions.

**Sample problem:**

1) Crystal Line is pushing the box across the floor with a constant force and the box moves at a speed of 1.0 m/s.

Answer the following questions about the forces acting on the box.

a) What forces are acting on the box? How do you know?

1. Crystal pushing the box to the right. I was told in the problem.

2. The floor pushes the box to the left. If there was no force pushing to the left, Crystal’s force would be the only horizontal force and the box would not move at a constant speed because the net force could not be zero.

3. The Earth is pulling the box down. The earth pulls everything on it (and close to it) down.

4. The floor pushing the box up. The box is not accelerating downward so there must be a force balancing the Earth’s pull downward.

b) Draw a complete force diagram for the forces acting on Crystal and the box. Indicate which forces, if any are Newton’s 3rd law pairs.



FNbC

FgEC



FNfC

These two are a 3rd law pair. Third Law pairs always have the same objects but in the opposite order. Crystal is pushing on the box to the right and the box is pushing her equally to the left. There is a third law pair for every force but they are not acting on Crystal or the box.

FNfb

FNCb

FgEb



c) What is the net horizontal force on the box? How do you know?

The net horizontal force is zero because the box is moving horizontally at a constant velocity.

d) What is the net vertical force on the box? How do you know?

The net vertical force is zero because the box is at rest vertically.

e) What is the reaction force to each force of the forces listed in a)?

1) The box is pushing Crystal to the left.

2) The box is pushing the floor to the right.

3) The box is pulling the Earth up.

**Tip**: When you do action/reaction questions, take the action statement, switch the position of the two objects around and change the direction to the opposite.

Action: The Earth is pulling the box down.

Reaction: The box is pulling the Earth up.

switch words around

change direction

4) The box is pushing the floor down.

2) The hand is causing the ball to accelerate upward at 2.0 m/s2.

a) What forces are acting on the ball?

The hand is pushing the ball up and the Earth is pulling the ball down.

FNhb

FgEb

b) Which of the two forces is greater? How do you know?

The hand force is greater. I know because the ball is accelerating upward. If the weight force were larger, the ball would be accelerating downward.

c) Draw a force diagram for the ball. (Done to the right.)

d) What is the net force on the ball?

The net force is the force of the hand minus the weight of the ball.

e) What is the reaction force to the weight of the ball?

Since the weight is caused by the Earth pulling the ball down, the reaction force is the ball pulling the Earth up. (If you are ever tempted to make the reaction force another force on the same object, don’t do it. Action and reaction forces are never acting on the same object.)

f) Which is greater, the force of the Earth pulling the ball down or the force of the ball pulling the Earth up?

The forces are exactly the same as stated in Newton’s third law.

g) If the two forces are equal, why doesn’t the Earth move upward when the ball pulls it?

The earth has so much more mass that the force causes very little acceleration. (Newton’s 2nd law)